MID-SEMSTER EXAMINATION
II SEMESTER, 2009-2010

B. MATH II YEAR
ANALYSIS IV

Time Limit: 2hrs Max Marks: 40

1. Let (X, d) be a metric space such that for any two points x and y there is a continuous map $f : [0, 1] \to X$ with $f(0) = x$ and $f(1) = y$. Prove that (X, d) is connected. *Do not assume connectedness of any particular space.* [8]

2. Prove or disprove:

a) An open ball in a metric space is necessarily connected.

b) An open ball in a normed vector space is necessarily connected. [6]

3. Let X be the space of all sequences of real numbers with the metric $d(\{a_n\}, \{b_n\}) = \sum_{n=1}^{\infty} \frac{|a_n - b_n|}{2^n[1+|a_n-b_n|]}$. Prove that the set $\{\{a_n\} : |a_n| \leq n \forall n\}$ is compact. [8]

4. Let $\{f_n\}$ be any sequence in $C[0, 1]$ and $g_n(x) = \int_0^x \tan^{-1}(f_n(t))dt$ Show that $\{g_n\}$ has a subsequence that converges in $C[0, 1]$ with the usual uniform metric. [8]

5. Let $Lip(\alpha) = \{f \in C[0, 1] : |f(x) - f(y)| \leq C |x - y|^{\alpha} \forall x, y\}$ where $\alpha > 0$. If $\alpha > 1$ prove that $Lip(\alpha)$ is the space of constant functions. Prove that there is a function in $Lip(\frac{1}{2})\backslash Lip(1)$. [10]